Quantifying control effort of biological and technical movements: an information entropy based approach

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Abstract

In biomechanics and biorobotics, muscles are often associated with reduced movement control effort and simplified control compared to technical actuators. This is based on evidence that the non-linear muscle properties positively influence movement control. It is, however, open how to quantify the simplicity aspect of control effort and compare it between systems. Physical measures, such as energy consumption, stability, or jerk, have already been applied to compare biological and technical systems. Here, a physical measure of control effort based on information entropy is presented. The idea is that control is simpler if a specific movement is generated with less processed sensor information – depending on the control scheme and the physical properties of the systems being compared. By calculating the Shannon information entropy of all sensor signals required for control, an information cost function can be formulated allowing the comparison of models of biological and technical control systems. Exemplarily applied to (bio-)mechanical models of hopping, the method reveals that the required information for generating hopping with a muscle driven by a simple reflex control scheme is only $I = 32\,\text{bits}$ vs. $I = 660\,\text{bits}$ with a DC-motor and a proportional differential (PD) controller. This approach to quantifying control effort captures the simplicity of a control scheme and can be used to compare completely different actuators and control approaches.

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I. INTRODUCTION

The elegance and diversity of biological movement is absolutely fascinating, and it is an inspiring challenge to investigate how animals and humans generate and control it. Biological movement emerges from the coordinated interaction of many subsystems and elements. Comprehensive scientific analysis in biology, physiology, neuroscience, and biophysics has lead to mathematical models predicting the functionality of many such subsystems and elements. Their interaction is then studied in biomechanical simulations with the goal to synthesize the complex behavior observed in nature and thus gain knowledge on their functional role [1–4]. Additionally, bio-inspired robots are important tools to verify the real world functionality of the theoretical concepts [5], and thus, play an important role in studying such systems [5–10]. The scientific value of such robotic systems requires a comparison to the biological role model. This comparison depends on valid quantification criteria. Several criteria known from physics have already been adapted and used for this purpose. Examples are performance measures such as running velocity [11] or jumping height [12], but also energy efficiency [13] or control stability [14]. Here, we propose a new method to quantify *control effort* based on information entropy.

Our approach is based on the general observation that biological systems generate and control movements with seemingly little effort, while it is still a challenge for robotic systems to imitate biological movements. If both biological and robotic systems are interpreted as control systems based on the cybernetic analogy between animals and machines [15], *control effort* in the sense above is associated with the effective interaction of actuator, controller, system, and environment. In classical control theory, systems with linear actuators, e.g. electric motors, have been well studied and many tools exist for generating effective controllers (e.g. proportional-differential PD) that satisfy different requirements with the existing trade offs being well documented too. By contrast, biological muscles have non-linear force production characteristics. Biomechanical models of muscle contraction dynamics support and emphasize that unique feature [1, 16–21]. Interestingly, it has been suggested that these very non-linearities of biological structures could reduce *control effort* and simplify control of biology-like movements [14, 22–30].

So far, this hypothesis has not been confirmed because previous quantitative definitions of *control effort* were not suitable to quantify the aspect of control simplicity and compare it between biological and technical systems. The term control effort is, in fact, associated with many different physical quantities. Some studies in engineering define control effort simply as a signal in time. Examples are output signal voltage of a controller [31, 32], motor armature voltage [33], actuation voltage in polymer actuators [34], or pressure in pneumatic actuators [35]. In a similar way, muscle electromyography (EMG) signal is associated with control effort in a neurological feedback control model [36]. With this signal based approach, control effort cannot be compared between biological and technical systems — it only allows for a qualitative comparison of different controllers in the same system. A quantitative approach is to derive a single value from a certain function of control to represent control effort, e.g., the maximum, the mean, or the integral of a signal over time [37–40]. The resulting values can be interpreted as cost functions to be minimized in controller optimizations, e.g., to minimize joint torques [38–40], actuator energy consumption [38, 39], movement jerk [38, 41], or muscle activity [2, 42]. Depending on the chosen function of control (signal), it is then possible to quantitatively compare entirely different control systems performing the same task. An example would be the total energy consumed to run one hundred meters.

To quantify the simplicity aspect of control, we propose to measure *information*. In our interpretation, control of one cybernetic system is simpler if the *information* required and processed by its controller is less than in another system. As pointed out by Touchette & Lloyd [43], controllers can be interpreted as communication channels in the sense that they transform a control system from an initial state to a desired target state. To do so, information on the system state must be constantly acquired by sensors and processed by the controller [44]. We hypothesize that the amount of required information to perform a specified task depends on the system's (bio-)physical properties, e.g., the muscle's non-linear properties allow to perform biological movements with less information on the system state as compared to linear actuators such as DC-motors. To test this hypothesis, a method is required allowing to quantify and compare the processed information in biological and technical control systems.

In this article, we propose to quantify the processed information in a control system by applying Shannon's information entropy to the sensor signals required for control. This method returns a single value for each system performing a specified task. It represents a physics based measure which allows to compare structurally different realizations of the same movement and thus allows to compare the control effort of technical and biological control systems.

II. METHOD: INFORMATION IN SENSOR MEASUREMENTS

The approach is based on the cybernetic analogy between humans and machines describing both as control systems. To achieve a goal, both use sensors to take measurements of their state and the environment, transmit and process the gathered information, and take actions accordingly [15, 45]. But how can the amount of processed information be quantified? According to Shannon [46], the prior uncertainty of the outcome when measuring the variable u can be quantified as the entropy of an information source (e.g., a sensor):

$$H(u) = -K \sum_{i=1}^{n} p_i \log_2 p_i \quad , \tag{1}$$

where $p_i = p(u = u_i)$ is the probability of the specific sensor measurement result $u = u_i$, with $\Sigma p_i = 1$. The constant $K = 1\,\text{bit}$ defines the unit of information.

Let us consider a linear sensor measuring a state variable u of a system. The sensor has a range $u_{\min} \leq u \leq u_{\max}$ and a resolution Δu and, thus, $n = 1 + (u_{\max} - u_{\min})/\Delta u$ possible measurement results. In each measurement j, it will measure one value of the values $u_i = u_{\min} + (i-1)\Delta u$, $i = 1 \ldots n$ with the probability of p_i. The information gained in each measurement I_j is

$$I_j = -\sum_{i=1}^{n} p_{ji} \log_2 p_{ji} \quad , \tag{2}$$

taking possible changes of the probability distribution between measurements into account.

In a continuous movement, the measurement is done once for each time step t_j, with $j = 1 \cdots m$, until the goal is reached at t_m. The total information processed in the task is

$$I = -\sum_{j=1}^{m} \sum_{i=1}^{n} p_{ji} \log_2 p_{ji} \quad . \tag{3}$$

For a real cybernetic system, the probabilities p_{ji} are a priori unknown and difficult to determine. In a typical sensor implementation, however, there is no prior assumption about the measurement outcome on the controller side. The sensor value is directly transmitted to the controller with its full possible range of output values. Although some values may never be reached, the controller allows their theoretical existence. Thus, each sensor value is equally likely from the controller's point of view $p_i = 1/n$. As a consequence, all sensor values are equally likely in each measurement: $p_{ji} = 1/n$. With this simplification, the total information is

$$I = -m \log_2 \frac{1}{n} = m \log_2 n \quad . \tag{4}$$

Eq. 4 can be applied to almost any type of sensor. It requires only a discretized sensor output and finite number of repeated measurements. The information can then be determined from the duration of the movement T, the time resolution Δt, and the sensor properties:

$$I = \frac{T}{\Delta t} \log_2 \left(1 + \frac{u_{\max} - u_{\min}}{\Delta u}\right) \quad . \tag{5}$$

This measure can be directly applied to most technical control systems. Furthermore, it can be applied to computer simulations of technical and biological control systems. To calculate the control effort, Shannon entropy has to be determined for all signals contributing to the control. Physically, they represent the signals that need to be measured by sensors, conducted via nerves or cables, and processed to change the active force of the actuator. To allow the calculation of I, these signals therefore have to be sampled with a defined time and amplitude resolution. Numerical variables that represent physical/material properties (e.g. elongation to calculate elastic forces, or motor velocity for the back electro-magnetic-force) do not contribute to the control effort and must not be discretized.

The choice of the resolutions Δu and ΔI has a major influence on the outcome of I. However a fair comparison of different control schemes can be achieved by optimizing the resolutions to identify the minimally required information $I_{\min}$ for each control scheme. If a control scheme works with coarse signal resolution and therefore little processed information, the control effort is low and the control is simple. The proposed steps for the comparison of different control systems for the same movement are therefore:

Step 1: Define a desired movement with a performance measure P.

Step 2: Model different control systems which generate the desired movement. Utilize continuous (or very high resolution) control signals and determine a reference performance ($\mathrm{P}_{\max}$).

Step 3: Discretize all signals contributing to the control with resolutions Δu and Δt. Optimize Δu and Δt for minimal information $I_{\min}$, while considering that the performance does not drop below a defined threshold (e.g., $P \geq 0.9P_{max}$). This assures that the control still generates the desired movement despite the low signal resolution.

III. APPLICATION TO MODELS FOR HOPPING

As an example, we apply the proposed measure (Eq. 5) to biological and technical models of periodic hopping. Human hopping was chosen as it is an easy to define one-dimensional motion which can be generated by many different actuation and control methods. Furthermore, it is a movement primitive of legged locomotion and as such biologically and evolutionarily relevant.

In its simplest form, hopping can be described by the following differential equation [26]:

$$M\ddot{y} = -Mg + \begin{cases} 0 & y > l_0 \quad \text{flight phase} \\ F_{\text{L}} & y \leq l_0 \quad \text{ground contact} \end{cases} . \tag{6}$$

Here, the body of the hopper is idealized as a point mass M which is accelerated by gravity in negative y-direction and during ground contact ($y \leq l_0$) by the leg force F_{L} in positive y-direction (see Table I for parameters). For periodic hopping, the leg force F_{L} has to generate alternating stance and flight phases. Three models for the leg force F_L were investigated. Two muscle models (previously published) and one DC-motor model with appropriate (neural) controllers were implemented and the respective information (Eq. 5) was calculated. Here, we explain the modifications to the models necessary to calculate the information. More details on the biomechanical models are given elsewhere [26, 47].

Step 1: Define the desired movement. The desired movement was stable periodic hopping (Eq. 6) with high ground clearance $h = y_{\text{max}} - l_0$ at frequencies $f > 2\,\text{Hz}$ ($T < 0.5\,\text{s}$). In accordance with this definition, stability of periodic hopping was evaluated with Poincaré-map analysis [48], and control performance P was determined as [26]:

$$\text{P} = h \cdot c \quad \text{with } c = \begin{cases} 1 & f \geq 2\,\text{Hz} \\ 20\,\text{s}^{-1}(0.55\,\text{s} - \frac{1}{f}) & f < 2\,\text{Hz} \end{cases} , \tag{7}$$

effectively reducing performance for frequencies $f < 2\,\text{Hz}$.

Step 2: Model different control systems.

Model MFF: Muscle model with feed-forward control strategy. In the MFF model, the leg force (in Eq. 6) was generated by one model muscle representing the net properties of all major leg muscles. The muscle's force F_M was

$$F_L = F_M = A(t) F_{\text{mat}}(l_M, \dot{l}_M) \ , \tag{8}$$

where $0 \leq A(t) \leq 1$ is the control parameter for the force. It represents the muscle activity, with $A = 1$ corresponding to a fully active muscle. F_{mat} is a function of muscle length l_M and muscle contraction velocity $\dot{l}_M$,

$$F_{\text{mat}} = \exp\left(\text{-}c \left| \frac{l_M - l_{\text{opt}}}{l_{\text{opt}} w} \right|^3 \right) \cdot \begin{cases} \frac{\dot{l}_{M,\max} + \dot{l}_M}{\dot{l}_{M,\max} - K \dot{l}_M} & v > 0 \\ N + (N-1) \frac{\dot{l}_{M,\max} - \dot{l}_M}{-7.56 K \dot{l}_M - \dot{l}_{M,\max}} & v \leq 0 \end{cases} \tag{9}$$

representing the muscle fibers' material properties [49] known to be important for hopping control [26, 47, 50, 51]. The first term is called the force-length relation, the second the force-velocity relation. A detailed description and motivation of the material model F_{mat} and its parameters (Table I) can be found in [26]. In previous studies [26, 47], we varied F_{mat} and analyzed the influence on hopping stability. Here, we chose the muscle model which resulted in the most stable hopping pattern, which also is the model representing the muscle properties most realistically (non-linear phenomenological model M[Hill, Hill], see Equations (3) and (4), and Figure 2 in [26]). This model does not consider any leg geometry nor elastic structures such as tendons. It is therefore the simplest hopping model taking into account the muscles' dynamic properties and allowing the application of different control strategies.

In the MFF model, we used a feed-forward control strategy. For the analysis of the control effort, all signals contributing to the control have to be quantified according to Eq. 5. These are a feed-forward signal and a trigger signal. The feed-forward signal can be interpreted as a learned activity pattern represented by a memorized time-series. For this signal, $u_{\min} = 0$ and $u_{\max} = 1$ stand for the minimal and maximal muscle activity, respectively. The amplitude resolution was $\Delta u = \Delta \text{A}$ und time resolution was $\Delta t = \Delta t_{\text{pattern}}$. The second signal required for the control was the detection of the take-off event (stance phase $\rightarrow$ flight phase). This event triggers the feed-forward pattern. The take-off sensor was represented by a boolean signal where $u_{\max} = 1$ applies only if take-off occurs and $u_{\min} = 0$ in all other

Table I. Model parameters for human hopping. The biological relevance of these parameters is motivated in [26].

	parameter	value
general	leg rest length l_0	1 m
general	body mass M	80 kg
general	gravitational constant g	$9.81\,\mathrm{ms^{-2}}$
muscle F_{mat}	maximum isometric muscle force $F_{\max}$	2.5 kN
muscle F_{mat}	optimal muscle length l_{opt}	0.9 m
muscle F_{mat}	width w	0.45 m
muscle F_{mat}	curvature c	30
muscle F_{mat}	maximum velocity $\dot{l}_{\max}$	$-3.5\,\mathrm{ms^{-1}}$
muscle F_{mat}	curvature constant K	1.5
muscle F_{mat}	eccentric force enhancement N	1.5

measurements. The amplitude resolution therefore was $\Delta u = 1$ and it was measured at time intervals of $\Delta t_{\text{take-off}}$.

The total information required for generating hopping in this control approach is the sum of the information content of the feed-forward pattern and the information processed to measure the take-off event:

$$I = \frac{T}{\Delta t_{\text{pattern}}} \log_2 \left(1 + \frac{1-0}{\Delta A}\right) + \frac{T}{\Delta t_{\text{take-off}}} \log_2 \left(1 + \frac{1-0}{1}\right) \quad . \tag{10}$$

Model MFB: Muscle model with direct feedback control strategy [47]. The leg force in the MFB model was generated by the same muscle model, i.e. the same material properties F_{mat}, as in the MFF model (Eq. 8, [26]). However, the control approach for generating $A(t)$ differed. Here, the muscle activity $A(t)$ was based on a feedback signal encoding the muscle force. This is, to our knowledge, the simplest hopping model with a representation of biological reflex pathways, i.e. the muscle activity being modulated by propriozeptive signals. For this purpose, the continuous muscle force $F_M(t)$ was sampled with an amplitude resolution Δu and a time resolution Δt to represent the sensor signal

$$u(t) = \text{round}\left(\frac{F_M(j\Delta t)}{F_{M,\max}\Delta u}\right)\Delta u \quad \text{for}\, j\Delta t \le t < (j+1)\Delta t \tag{11}$$

with $j = 1 \cdots m$, $m = T/\Delta t$, and $F_{M,\max}$ being the maximum muscle force. The muscle activity $A(t)$ was then calculated from the delayed sensor signal $u(t-\delta)$ with a first-order differential equation taking into account the gross time-behavior of the chemical processes that lead from neural stimulation to muscle force [47, 50]

$$\frac{dA(t)}{dt} = \frac{1}{\tau}\left(G \cdot u(t-\delta) + A_0 - A(t)\right) \quad . \tag{12}$$

G is a gain factor for the feedback signal, $\delta = 15\,\mathrm{ms}$ the feedback delay due to neural signal latency, $\tau = 10\,\mathrm{ms}$ is the typical time constant of the chemical processes, and $A_0 = 0.03$ represents the muscle activity at touch-down.

This control approach represents a direct force-feedback control modulating the muscle force (Eq. 8) while considering also the time-behavior of the chemical processes in the muscle. The only signal contributing to the control is the muscle force signal, for which $u_{\min} = 0, u_{\max} = 1$. With the resolutions Δu and Δt, I can be directly calculated using Eq. 5.

Model EFB: Electric DC-motor model with proportional-differential (PD) feedback control. To compare the biomechanical model of hopping to a technical control approach, a DC-motor-driven model was implemented. Here, the leg force F_L (Eq. 6) was modeled as

$$F_L = \gamma T_L = \gamma k_T I_{DC} \tag{13}$$

where k_T is the motor constant, I_{DC} the current through the motor windings, and γ the ratio of an ideal gear translating the rotational torque T_L and movement $\varphi(t)$ of the motor to the translational leg force and movement required for hopping. The electrical characteristics of the motor can be modeled as

$$\dot{I}_{DC} = \frac{1}{L}\left(U_a - k_T \dot{\varphi} - R I_{DC}\right) \tag{14}$$

where U_a is the armature voltage, R the resistance, and L the inductance of the motor windings. The motor parameters, including the maximally allowed voltage of $U_{a,\max} = 48\,\mathrm{V}$, were taken from a commercially available DC-motor commonly used in robotics applications (Maxon EC-max 40, $k_T = 0.126\,\mathrm{Nm/A}$, $R = 7.19\,\Omega$, $L = 1.6\,\mathrm{mH}$). γ was chosen such that the maximum leg forces were comparable to those of the muscle models which allow hopping with comparable performance P.

To generate periodic hopping, a negative feedback control scheme was used to enforce a desired kinematic trajectory $y_{\mathrm{des}}(t)$ during ground contact. The idea of negative feedback control is to measure the current position $y(t)$, compare it to the desired position $y_{\mathrm{des}}(t)$ and control the motor such that the error $e = y_{\mathrm{des}}(t) - y(t)$ is minimal. As desired trajectory, the hopping pattern from the MFB model with high resolution was used. The control input to the motor model is the armature voltage U_a, which was adjusted by a PD-controller

$$\begin{aligned} U_a(t) &= G_P(\frac{1}{\gamma}y_{\mathrm{des}}(t) - \frac{1}{\gamma}y(t)) + G_D(\frac{1}{\gamma}\dot{y}_{\mathrm{des}}(t) - \frac{1}{\gamma}\dot{y}(t)) \\ &= G_P e(t) + G_D \dot{e}(t) \end{aligned} \tag{15}$$

to minimize e.

The signals, contributing to this type of control are the feedback signals position $y(t)$ and velocity $\dot{y}(t)$, and the desired time series for $y_{\mathrm{des}}(t)$, and $\dot{y}_{\mathrm{des}}(t)$. The sensor limits for these signals were taken from the original desired trajectories as $u_{y,\min} = u_{y_{\mathrm{des}},\min} = \min(y_{\mathrm{des}}(t))$ and $u_{y,\max} = u_{y_{\mathrm{des}},\max} = \max(y_{\mathrm{des}}(t))$, and in an analogous manner for the velocity signals. All four signals were encoded with the same time resolution Δt and amplitude resolutions $\Delta u_y = \Delta u_{y_{\mathrm{des}}} = (u_{y,\max} - u_{y,\min})/n$, and $\Delta u_{\dot{y}} = \Delta u_{\dot{y}_{\mathrm{des}}} = (u_{\dot{y},\max} - u_{\dot{y},\min})/n$, respectively. The total processed information I can thus directly be calculated by four addends of type Eq. 5:

$$\begin{aligned} I =& \frac{T}{\Delta t}\left(2\log_2\left(1 + \frac{u_{y,\max} - u_{y,\min}}{\Delta u_y}\right)\right. \\ & \left. +2\log_2\left(1 + \frac{u_{\dot{y},\max} - u_{\dot{y},\min}}{\Delta u_{\dot{y}}}\right)\right) \quad . \end{aligned} \tag{16}$$

This model is the simplest implementation of negative feedback control that allows to enforce a desired hopping trajectory on a technical system.

Step 3: Find $I_{\min}$ by optimizing the signal resolution parameters. By definition, the information I depends on the chosen resolutions Δt and Δu (Eq. 5). To adequately compare the three models, the minimally required information $I_{\min}$ for stable hopping was determined by optimization. For this purpose, hopping was first performed with high resolutions (equivalent to large I) to determine a reference performance $\mathrm{P}_{\max}$. The reference performance was verified with sensor resolutions one magnitude higher, which resulted in the same performance $P_{\max}$. Then, the resolutions, and thus I, were systematically decreased. For each resolution, the control parameters (feed-forward pattern $A(t)$, or feedback parameters G, or G_P and G_D, respectively) were optimized for maximum stable hopping performance. $I_{\min}$ was chosen as the lowest resolution resulting in stable hopping with a hopping performance of at least $P_{I_{\min}} > 0.9P_{\max}$. To gain an estimate of the error of $I_{\min}$, the difference $\Delta I = I_{\min -1} - I_{\min}$ to the second lowest resolution found in the optimization was calculated. ΔI thus gives an estimate for a possible reduction in $I_{\min}$ if the search of the optimization would be further refined. The optimization step size was chosen such that the error $\Delta I \approx 10\% I_{\min}$. A corresponding error was calculated for the resolutions and control parameters. These errors specify the steps by which the values were varied during the optimization.

In the MFF model, first the time-series $A(t)$ was optimized for hopping performance P (Eq. 7) on grids with different resolutions ΔA and $\Delta t_{\text{pattern}}$ (algorithm described in [26]). The pattern with the lowest encoded information was found to have resolutions of $\Delta t_{\text{pattern}} = 0.125 \pm 0.025\,\text{s}$ and $\Delta A = 0.125 \pm 0.025$, therefore $I_{\text{pattern}} = 12 \pm 3\,\text{bits}$. Afterwards, the minimal required time resolution of the trigger event detection was searched ($\Delta t_{\text{take-off}} = 0.02\,\text{s}$). The minimal required information to generate hopping in this model was thus found to be $\mathbf{I}_{\min} = (\mathbf{34 \pm 3})\,\mathbf{bits}$. The achieved hopping performance at $I_{\min}$ was $P_{I_{\min}} = 0.062\,\text{m} = 91\% \cdot P_{\max}$. Further reduction of the resolution resulted in frequencies $f < 2\,\text{Hz}$ and thus reduced performance $P < 90\% \cdot P_{\max}$.

In the MFB model, the information to be transmitted from the force sensor to the muscle only depends on the signal resolution parameters Δu and Δt. Their optimization resulted in $\mathbf{I}_{\min} = (\mathbf{32 \pm 2})\,\mathbf{bits}$. The corresponding resolutions of the feedback-signal were $\Delta t_{I_{\min}} = 0.0650 \pm 0.0025\,\text{s}$ and $\Delta u_{I_{\min}} = 0.0525 \pm 0.0025$. The control parameter feedback gain was $G = 9.43 \pm 0.24$. The achieved performance was $P_{I_{\min}} = 0.068\,\text{m} = 96\% \cdot P_{\max}$. As in the MFF model, further reduction of the resolution resulted in frequencies $f < 2\,\text{Hz}$ and thus reduced performance $P < 90\% \cdot P_{\max}$.

In the EFB model, the minimally required information was found to be $\mathbf{I}_{\min} = (\mathbf{660 \pm 63})\,\mathbf{bits}$. The corresponding resolutions of the PD-controller were $\Delta t_{I_{\min}} = 0.0082 \pm 0.001\,\text{s}$, $\Delta u_{y,I_{\min}} = 0.024 \pm 0.004$, and $\Delta u_{\dot{y},I_{\min}} = 0.49 \pm 0.08$. The optimal feedback gains were $G_P = (6.250 \pm 0.25)10^3\,\text{Vm}^{-1}$ and $G_D = (1 \pm 0.1)10^3\,\text{Vsm}^{-1}$ resulting in a hopping performance of $P_{I_{\min}} = 0.067\,\text{m} = 95\% \cdot P_{\max}$. In this model further reduction of the resolution results in unstable behavior rather than a reduction in performance.

IV. DISCUSSION

We have presented a new method to quantify *control effort.* Despite – or more likely because of – its simple form, it can be directly applied to real technical control systems, but also to models of biological and technical control systems by discretizing all (virtual) sensor signals contributing to the control. The foundation, Shannon's information entropy, is not new. Since its publication in 1948 [46], it has been applied in many fields. Its general formulation (Eq. (1)) is based on the probabilities $p(x)$ of certain signals, code words, etc. The application of this general form to real or modeled control systems is often difficult, as these probabilities are unknown a priori. With the assumption of equal probability for each sensor value, the application becomes possible (Eq. (4)). Using this simplification, our approach takes advantage of the fact that information entropy, applied to sensor measurements, gives a numerical value for the processed information. This value is – in the sense of optimal control [38] – a cost function allowing to quantify, compare, and optimize systems. The novelty of our approach is to measure the information entropy of the movement control process as a function of physical structure and control method:

$$I_{\text{movement}} = I_{\text{movement}}(\text{structure},\ \text{control}) \tag{17}$$

The chosen examples demonstrate this. All three models MFF, MFB, and EFB execute the same motion, i.e. periodic hopping. The models are of similar reduced complexity with only one actuator and the minimum number of control signals required. Thus, the results are comparable. Furthermore, the muscle models take important non-linear characteristics of biological muscles into account and rely on simple bio-inspired control schemes, while the technical model implements a standard technical actuator in combination with a standard feedback controller to generate a desired hopping trajectory. Between muscle models MFF and MFB, only the control is varied while the bio-physical structure, i.e. the muscle fibers' material properties, is the same (Eq. (17)). For the electric motor model EFB, additionally the physical structure is varied. The resulting values for the minimally required information reveal that the muscle requires less control effort for hopping as it requires considerably less information to generate and stabilize the movement (MFF: $I_{\min} = 34\,\text{bits}$, or MFB: $I_{\min} = 32\,\text{bits}$), compared to the engineering approach (EFB: $I_{\min} = 660\,\text{bits}$). This confirms that the requirements on information processing – or *cognitive load* – depends on the (bio-)physical properties of a control system [52].

In contrast to other studies [53, 54], our approach only quantifies the information leading from sensor signals to actuator actions while ignoring the information back-flow via system dynamics and environment. Within the sensor signal lies the relevant information for the controller to generate the desired movement, also called *pragmatic information* [55]. Pragmatic information is only the information that actually generates a measurable action or change in structure stripped of all the redundant and unnecessary information. The assumption of equal distribution ($p_i = 1/n$) results in the upper bound of the information compared to all other possible distributions ($p_i \neq 1/n$) [56]:

$$0 \leq -\sum_{i=1}^{n} p_i \log_2 p_i \leq \log_2 n \enspace .$$

Therefore, our approach typically overestimates the transmitted information. The optimization with I as cost function to be minimized by varying sensor resolutions is a way to approximate the pragmatic information of the sensor signal and thus allowing a comparison of different realizations of the same movement — the lower I_{min} is, the lower is the pragmatic information while I_{min} with $p_i = 1/n$ gives its upper bound.

The more general approach to include the flow of information back to its sensors, that the actuated system causes via system dynamics and environment, additionally allows to investigate optimal principles on the decisions which movement to perform [53, 54]. At the moment, our approach only targets different realizations of the same movement and focuses on the differences resulting from the (bio-)physical properties of the system. For this purpose, it is required to implement the method in concrete models of control systems with the trade-off on the generality of the conclusions. Also, the simplifying assumption of equal distribution only approximates information theoretic limits. More general conclusions on information theoretic limits of open-loop (feed-forward) vs. closed-loop (feedback) control can only be drawn if the models' explicit (bio-)physical properties are not considered and the probability distribution of the sensor readings are not restricted [43, 44]. Future work has to reveal the relation of our approximation to the information theoretic limits in the control loop.

Our method to quantify control effort relies on the explicit definition of the desired movement and a corresponding definition of the movement performance (here, Eq. 7). Stable hopping is quite suitable for this purpose. The trade-off is, however, that the results are only valid for just the investigated movement. It is expected that the results may vary quite substantially for other movements . This limitation to general conclusions about control systems is also known from other quantification criteria, e.g. energy requirements, which are movement specific too. The reduced examples on hopping therefore primarily demonstrate the application of our proposed measure. To confirm in general the hypothesis that muscles reduce control effort compared to technical actuators in the control of biology-like movements, more models, movements, and controllers need to be compared.

Nevertheless, we expect that more complex biomechanical models will confirm the low control effort requirements. The material properties of the actuator in the muscle driven models MFF and MFB represent typical properties of biological muscles, i.e. the well-known muscle force-length-velocity dependency. These reduced models were specifically introduced to reveal the relevance of muscle properties in periodic movements, such as hopping [26, 47] and confirmed previous findings that the force-length-velocity relation of muscles is significant with respect to the control of biological movements [1, 26, 50, 51, 57, 58]. More precisely, neglecting the force-velocity relation results in unstable hopping with the proposed simple bio-inspired controllers in the reduced [26, 47] and in more complex models [50, 51]. We therefore expect that also the tendency for little required control effort will be inherited by more detailed models of human hopping. The reason for this expectation is the concept of *exploitive actuation* [5]. If the mechanical system is well designed, part of the control can be attributed to the mechanical system itself and thus be exploited by simple controllers. There is evidence that biological systems are designed in such a way [5, 24, 52, 59–62]. A technical solution which is not specifically optimized with respect to this concept may require significantly larger control effort. However, we expect that also technical solutions can be found which result in a desired movement with minimal control effort. There are even technical solutions that require less control effort than their biological role model. For example, passive dynamic walkers and runners, which perform stable biology-like locomotion without any actuation [63–65] and therefore with zero control effort ($I_{\mathrm{movement}} = 0$). However, passive dynamic walkers or runners can only perform one specific movement. In contrast, humans can perform a large variety of movements, including walking and running. This demonstrates that control effort calculated by information entropy may prove to be an important measure in the study of biological systems. As opposed to previous definitions of control effort, it captures the simplicity aspect of movement control. It is therefore one additional quantitative measure, besides e.g. stability, consumed energy, time to complete a task, minimum jerk, performance, and other possible cost functions, allowing a quantitative comparison of structurally different realizations of the same movement. Further research has to reveal whether minimal processed information is a design principle in nature as some studies suggest [66].

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