

1 **Quantifying control effort of biological and technical movements:**
2 **an information entropy based approach**

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Abstract

In biomechanics and biorobotics, muscles are often associated with reduced movement control effort and simplified control compared to technical actuators. This is based on evidence that the non-linear muscle properties positively influence movement control. It is, however, open how to quantify the simplicity aspect of control effort and compare it between systems. Physical measures, such as energy consumption, stability, or jerk, have already been applied to compare biological and technical systems. Here, a physical measure of control effort based on information entropy is presented. The idea is that control is simpler if a specific movement is generated with less processed sensor information – depending on the control scheme and the physical properties of the systems being compared. By calculating the Shannon information entropy of all sensor signals required for control, an information cost function can be formulated allowing the comparison of models of biological and technical control systems. Exemplarily applied to (bio-)mechanical models of hopping, the method reveals that the required information for generating hopping with a muscle driven by a simple reflex control scheme is only $I = 32$ bits vs. $I = 660$ bits with a DC-motor and a proportional differential (PD) controller. This approach to quantifying control effort captures the simplicity of a control scheme and can be used to compare completely different actuators and control approaches.

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15 I. INTRODUCTION

16 The elegance and diversity of biological movement is absolutely fascinating, and it is an
17 inspiring challenge to investigate how animals and humans generate and control it. Biological
18 movement emerges from the coordinated interaction of many subsystems and elements.
19 Comprehensive scientific analysis in biology, physiology, neuroscience, and biophysics has
20 lead to mathematical models predicting the functionality of many such subsystems and
21 elements. Their interaction is then studied in biomechanical simulations with the goal
22 to synthesize the complex behavior observed in nature and thus gain knowledge on their
23 functional role [1–4]. Additionally, bio-inspired robots are important tools to verify the
24 real world functionality of the theoretical concepts [5], and thus, play an important role
25 in studying such systems [5–10]. The scientific value of such robotic systems requires a
26 comparison to the biological role model. This comparison depends on valid quantification
27 criteria. Several criteria known from physics have already been adapted and used for this
28 purpose. Examples are performance measures such as running velocity [11] or jumping
29 height [12], but also energy efficiency [13] or control stability [14]. Here, we propose a new
30 method to quantify *control effort* based on information entropy.

31 Our approach is based on the general observation that biological systems generate and
32 control movements with seemingly little effort, while it is still a challenge for robotic systems
33 to imitate biological movements. If both biological and robotic systems are interpreted as
34 control systems based on the cybernetic analogy between animals and machines [15], *control*
35 *effort* in the sense above is associated with the effective interaction of actuator, controller,
36 system, and environment. In classical control theory, systems with linear actuators, e.g. elec-
37 tric motors, have been well studied and many tools exist for generating effective controllers
38 (e.g. proportional-differential PD) that satisfy different requirements with the existing trade
39 offs being well documented too. By contrast, biological muscles have non-linear force pro-
40 duction characteristics. Biomechanical models of muscle contraction dynamics support and
41 emphasize that unique feature [1, 16–21]. Interestingly, it has been suggested that these
42 very non-linearities of biological structures could reduce *control effort* and simplify control
43 of biology-like movements [14, 22–30].

44 So far, this hypothesis has not been confirmed because previous quantitative definitions
45 of *control effort* were not suitable to quantify the aspect of control simplicity and compare it

46 between biological and technical systems. The term control effort is, in fact, associated with
47 many different physical quantities. Some studies in engineering define control effort simply as
48 a signal in time. Examples are output signal voltage of a controller [31, 32], motor armature
49 voltage [33], actuation voltage in polymer actuators [34], or pressure in pneumatic actuators
50 [35]. In a similar way, muscle electromyography (EMG) signal is associated with control
51 effort in a neurological feedback control model [36]. With this signal based approach, control
52 effort cannot be compared between biological and technical systems — it only allows for a
53 qualitative comparison of different controllers in the same system. A quantitative approach
54 is to derive a single value from a certain function of control to represent control effort,
55 e.g., the maximum, the mean, or the integral of a signal over time [37–40]. The resulting
56 values can be interpreted as cost functions to be minimized in controller optimizations, e.g.,
57 to minimize joint torques [38–40], actuator energy consumption [38, 39], movement jerk
58 [38, 41], or muscle activity [2, 42]. Depending on the chosen function of control (signal), it
59 is then possible to quantitatively compare entirely different control systems performing the
60 same task. An example would be the total energy consumed to run one hundred meters.

61 To quantify the simplicity aspect of control, we propose to measure *information*. In our
62 interpretation, control of one cybernetic system is simpler if the *information* required and
63 processed by its controller is less than in another system. As pointed out by Touchette
64 & Lloyd [43], controllers can be interpreted as communication channels in the sense that
65 they transform a control system from an initial state to a desired target state. To do so,
66 information on the system state must be constantly acquired by sensors and processed by
67 the controller [44]. We hypothesize that the amount of required information to perform a
68 specified task depends on the system’s (bio-)physical properties, e.g., the muscle’s non-linear
69 properties allow to perform biological movements with less information on the system state
70 as compared to linear actuators such as DC-motors. To test this hypothesis, a method
71 is required allowing to quantify and compare the processed information in biological and
72 technical control systems.

73 In this article, we propose to quantify the processed information in a control system by
74 applying Shannon’s information entropy to the sensor signals required for control. This
75 method returns a single value for each system performing a specified task. It represents
76 a physics based measure which allows to compare structurally different realizations of the
77 same movement and thus allows to compare the control effort of technical and biological

78 control systems.

79 II. METHOD: INFORMATION IN SENSOR MEASUREMENTS

80 The approach is based on the cybernetic analogy between humans and machines describ-
81 ing both as control systems. To achieve a goal, both use sensors to take measurements of
82 their state and the environment, transmit and process the gathered information, and take
83 actions accordingly [15, 45]. But how can the amount of processed information be quanti-
84 fied? According to Shannon [46], the prior uncertainty of the outcome when measuring the
85 variable u can be quantified as the entropy of an information source (e.g., a sensor):

$$H(u) = -K \sum_{i=1}^n p_i \log_2 p_i , \quad (1)$$

86 where $p_i = p(u = u_i)$ is the probability of the specific sensor measurement result $u = u_i$,
87 with $\sum p_i = 1$. The constant $K = 1$ bit defines the unit of information.

88 Let us consider a linear sensor measuring a state variable u of a system. The sensor
89 has a range $u_{\min} \leq u \leq u_{\max}$ and a resolution Δu and, thus, $n = 1 + (u_{\max} - u_{\min})/\Delta u$
90 possible measurement results. In each measurement j , it will measure one value of the values
91 $u_i = u_{\min} + (i - 1)\Delta u$, $i = 1 \dots n$ with the probability of p_i . The information gained in each
92 measurement I_j is

$$I_j = - \sum_{i=1}^n p_{ji} \log_2 p_{ji} , \quad (2)$$

93 taking possible changes of the probability distribution between measurements into account.

94 In a continuous movement, the measurement is done once for each time step t_j , with
95 $j = 1 \dots m$, until the goal is reached at t_m . The total information processed in the task is

$$I = - \sum_{j=1}^m \sum_{i=1}^n p_{ji} \log_2 p_{ji} . \quad (3)$$

96 For a real cybernetic system, the probabilities p_{ji} are a priori unknown and difficult to
97 determine. In a typical sensor implementation, however, there is no prior assumption about
98 the measurement outcome on the controller side. The sensor value is directly transmitted
99 to the controller with its full possible range of output values. Although some values may
100 never be reached, the controller allows their theoretical existence. Thus, each sensor value

101 is equally likely from the controller's point of view $p_i = 1/n$. As a consequence, all sensor
 102 values are equally likely in each measurement: $p_{ji} = 1/n$. With this simplification, the total
 103 information is

$$I = -m \log_2 \frac{1}{n} = m \log_2 n . \quad (4)$$

104 Eq. 4 can be applied to almost any type of sensor. It requires only a discretized sensor output
 105 and finite number of repeated measurements. The information can then be determined from
 106 the duration of the movement T , the time resolution Δt , and the sensor properties:

$$I = \frac{T}{\Delta t} \log_2 \left(1 + \frac{u_{\max} - u_{\min}}{\Delta u} \right) . \quad (5)$$

107 This measure can be directly applied to most technical control systems. Furthermore, it can
 108 be applied to computer simulations of technical and biological control systems. To calculate
 109 the control effort, Shannon entropy has to be determined for all signals contributing to
 110 the control. Physically, they represent the signals that need to be measured by sensors,
 111 conducted via nerves or cables, and processed to change the active force of the actuator. To
 112 allow the calculation of I , these signals therefore have to be sampled with a defined time and
 113 amplitude resolution. Numerical variables that represent physical/material properties (e.g.
 114 elongation to calculate elastic forces, or motor velocity for the back electro-magnetic-force)
 115 do not contribute to the control effort and must not be discretized.

116 The choice of the resolutions Δu and ΔI has a major influence on the outcome of I .
 117 However a fair comparison of different control schemes can be achieved by optimizing the
 118 resolutions to identify the minimally required information I_{\min} for each control scheme. If a
 119 control scheme works with coarse signal resolution and therefore little processed information,
 120 the control effort is low and the control is simple. The proposed steps for the comparison of
 121 different control systems for the same movement are therefore:

122 **Step 1:** Define a desired movement with a performance measure P .

123 **Step 2:** Model different control systems which generate the desired movement. Utilize con-
 124 tinuous (or very high resolution) control signals and determine a reference performance
 125 (P_{\max}).

126 **Step 3:** Discretize all signals contributing to the control with resolutions Δu and Δt . Opti-
 127 mize Δu and Δt for minimal information I_{\min} , while considering that the performance

128 does not drop below a defined threshold (e.g., $P \geq 0.9P_{\max}$). This assures that the
 129 control still generates the desired movement despite the low signal resolution.

130 III. APPLICATION TO MODELS FOR HOPPING

131 As an example, we apply the proposed measure (Eq. 5) to biological and technical models
 132 of periodic hopping. Human hopping was chosen as it is an easy to define one-dimensional
 133 motion which can be generated by many different actuation and control methods. Fur-
 134 thermore, it is a movement primitive of legged locomotion and as such biologically and
 135 evolutionarily relevant.

136 In its simplest form, hopping can be described by the following differential equation [26]:

$$M\ddot{y} = -Mg + \begin{cases} 0 & y > l_0 & \text{flight phase} \\ F_L & y \leq l_0 & \text{ground contact} \end{cases} . \quad (6)$$

137 Here, the body of the hopper is idealized as a point mass M which is accelerated by gravity
 138 in negative y -direction and during ground contact ($y \leq l_0$) by the leg force F_L in positive y -
 139 direction (see Table I for parameters). For periodic hopping, the leg force F_L has to generate
 140 alternating stance and flight phases. Three models for the leg force F_L were investigated.
 141 Two muscle models (previously published) and one DC-motor model with appropriate (neu-
 142 ral) controllers were implemented and the respective information (Eq. 5) was calculated.
 143 Here, we explain the modifications to the models necessary to calculate the information.
 144 More details on the biomechanical models are given elsewhere [26, 47].

145 **Step 1:** Define the desired movement. The desired movement was stable periodic hopping
 146 (Eq. 6) with high ground clearance $h = y_{\max} - l_0$ at frequencies $f > 2$ Hz ($T < 0.5$ s). In
 147 accordance with this definition, stability of periodic hopping was evaluated with POINCARÉ-
 148 map analysis [48], and control performance P was determined as [26]:

$$P = h \cdot c \quad \text{with } c = \begin{cases} 1 & f \geq 2 \text{ Hz} \\ 20 \text{ s}^{-1} (0.55 \text{ s} - \frac{1}{f}) & f < 2 \text{ Hz} \end{cases} , \quad (7)$$

149 effectively reducing performance for frequencies $f < 2$ Hz.

150 **Step 2:** Model different control systems.

151 **Model MFF:** Muscle model with feed-forward control strategy. In the MFF model, the
 152 leg force (in Eq. 6) was generated by one model muscle representing the net properties of

153 all major leg muscles. The muscle's force F_M was

$$F_L = F_M = A(t)F_{\text{mat}}(l_M, \dot{l}_M) , \quad (8)$$

154 where $0 \leq A(t) \leq 1$ is the control parameter for the force. It represents the muscle activity,
 155 with $A = 1$ corresponding to a fully active muscle. F_{mat} is a function of muscle length l_M
 156 and muscle contraction velocity \dot{l}_M ,

$$F_{\text{mat}} = \exp\left(-c \left| \frac{l_M - l_{\text{opt}}}{l_{\text{opt}}w} \right|^3\right) \cdot \begin{cases} \frac{\dot{l}_{M,\text{max}} + \dot{l}_M}{\dot{l}_{M,\text{max}} - K\dot{l}_M} & v > 0 \\ N + (N - 1) \frac{\dot{l}_{M,\text{max}} - \dot{l}_M}{-7.56K\dot{l}_M - \dot{l}_{M,\text{max}}} & v \leq 0 \end{cases} \quad (9)$$

157 representing the muscle fibers' material properties [49] known to be important for hopping
 158 control [26, 47, 50, 51]. The first term is called the force-length relation, the second the
 159 force-velocity relation. A detailed description and motivation of the material model F_{mat}
 160 and its parameters (Table I) can be found in [26]. In previous studies [26, 47], we varied F_{mat}
 161 and analyzed the influence on hopping stability. Here, we chose the muscle model which
 162 resulted in the most stable hopping pattern, which also is the model representing the muscle
 163 properties most realistically (non-linear phenomenological model M[Hill, Hill], see Equations
 164 (3) and (4), and Figure 2 in [26]). This model does not consider any leg geometry nor elastic
 165 structures such as tendons. It is therefore the simplest hopping model taking into account
 166 the muscles' dynamic properties and allowing the application of different control strategies.

168

169 In the MFF model, we used a feed-forward control strategy. For the analysis of the control
 170 effort, all signals contributing to the control have to be quantified according to Eq. 5. These
 171 are a feed-forward signal and a trigger signal. The feed-forward signal can be interpreted as
 172 a learned activity pattern represented by a memorized time-series. For this signal, $u_{\text{min}} =$
 173 0 and $u_{\text{max}} = 1$ stand for the minimal and maximal muscle activity, respectively. The
 174 amplitude resolution was $\Delta u = \Delta A$ and time resolution was $\Delta t = \Delta t_{\text{pattern}}$. The second
 175 signal required for the control was the detection of the take-off event (stance phase \rightarrow flight
 176 phase). This event triggers the feed-forward pattern. The take-off sensor was represented
 177 by a boolean signal where $u_{\text{max}} = 1$ applies only if take-off occurs and $u_{\text{min}} = 0$ in all other

Table I. Model parameters for human hopping. The biological relevance of these parameters is motivated in [26].

	parameter	value
general	leg rest length l_0	1 m
	body mass M	80 kg
	gravitational constant g	9.81 ms^{-2}
muscle F_{mat}	maximum isometric muscle force F_{max}	2.5 kN
	optimal muscle length l_{opt}	0.9 m
	width w	0.45 m
	curvature c	30
	maximum velocity \dot{l}_{max}	-3.5 ms^{-1}
	curvature constant K	1.5
	eccentric force enhancement N	1.5

178 measurements. The amplitude resolution therefore was $\Delta u = 1$ and it was measured at time
179 intervals of $\Delta t_{\text{take-off}}$.

180 The total information required for generating hopping in this control approach is the
181 sum of the information content of the feed-forward pattern and the information processed
182 to measure the take-off event:

$$I = \frac{T}{\Delta t_{\text{pattern}}} \log_2 \left(1 + \frac{1-0}{\Delta A} \right) + \frac{T}{\Delta t_{\text{take-off}}} \log_2 \left(1 + \frac{1-0}{1} \right) . \quad (10)$$

183 **Model MFB:** Muscle model with direct feedback control strategy [47]. The leg force in
184 the MFB model was generated by the same muscle model, i.e. the same material properties
185 F_{mat} , as in the MFF model (Eq. 8, [26]). However, the control approach for generating
186 $A(t)$ differed. Here, the muscle activity $A(t)$ was based on a feedback signal encoding the
187 muscle force. This is, to our knowledge, the simplest hopping model with a representation
188 of biological reflex pathways, i.e. the muscle activity being modulated by proprioceptive
189 signals. For this purpose, the continuous muscle force $F_M(t)$ was sampled with an amplitude
190 resolution Δu and a time resolution Δt to represent the sensor signal

$$u(t) = \text{round} \left(\frac{F_M(j\Delta t)}{F_{M,\text{max}}\Delta u} \right) \Delta u \quad \text{for } j\Delta t \leq t < (j+1)\Delta t \quad (11)$$

191 with $j = 1 \dots m$, $m = T/\Delta t$, and $F_{M,\max}$ being the maximum muscle force. The muscle
 192 activity $A(t)$ was then calculated from the delayed sensor signal $u(t - \delta)$ with a first-order
 193 differential equation taking into account the gross time-behavior of the chemical processes
 194 that lead from neural stimulation to muscle force [47, 50]

$$\frac{dA(t)}{dt} = \frac{1}{\tau} (G \cdot u(t - \delta) + A_0 - A(t)) \quad . \quad (12)$$

195 G is a gain factor for the feedback signal, $\delta = 15$ ms the feedback delay due to neural signal
 196 latency, $\tau = 10$ ms is the typical time constant of the chemical processes, and $A_0 = 0.03$
 197 represents the muscle activity at touch-down.

198 This control approach represents a direct force-feedback control modulating the muscle
 199 force (Eq. 8) while considering also the time-behavior of the chemical processes in the muscle.
 200 The only signal contributing to the control is the muscle force signal, for which $u_{\min} =$
 201 $0, u_{\max} = 1$. With the resolutions Δu and Δt , I can be directly calculated using Eq. 5.

202 **Model EFB:** Electric DC-motor model with proportional-differential (PD) feedback
 203 control. To compare the biomechanical model of hopping to a technical control approach, a
 204 DC-motor-driven model was implemented. Here, the leg force F_L (Eq. 6) was modeled as

$$F_L = \gamma T_L = \gamma k_T I_{DC} \quad (13)$$

205 where k_T is the motor constant, I_{DC} the current through the motor windings, and γ the ratio
 206 of an ideal gear translating the rotational torque T_L and movement $\varphi(t)$ of the motor to the
 207 translational leg force and movement required for hopping. The electrical characteristics of
 208 the motor can be modeled as

$$\dot{I}_{DC} = \frac{1}{L} (U_a - k_T \dot{\varphi} - R I_{DC}) \quad (14)$$

209 where U_a is the armature voltage, R the resistance, and L the inductance of the motor
 210 windings. The motor parameters, including the maximally allowed voltage of $U_{a,\max} = 48$ V,
 211 were taken from a commercially available DC-motor commonly used in robotics applications
 212 (Maxon EC-max 40, $k_T = 0.126$ Nm/A, $R = 7.19 \Omega$, $L = 1.6$ mH). γ was chosen such that
 213 the maximum leg forces were comparable to those of the muscle models which allow hopping
 214 with comparable performance P.

215 To generate periodic hopping, a negative feedback control scheme was used to enforce a
 216 desired kinematic trajectory $y_{\text{des}}(t)$ during ground contact. The idea of negative feedback
 217 control is to measure the current position $y(t)$, compare it to the desired position $y_{\text{des}}(t)$ and
 218 control the motor such that the error $e = y_{\text{des}}(t) - y(t)$ is minimal. As desired trajectory,
 219 the hopping pattern from the MFB model with high resolution was used. The control input
 220 to the motor model is the armature voltage U_a , which was adjusted by a PD-controller

$$\begin{aligned}
 U_a(t) &= G_P\left(\frac{1}{\gamma}y_{\text{des}}(t) - \frac{1}{\gamma}y(t)\right) + G_D\left(\frac{1}{\gamma}\dot{y}_{\text{des}}(t) - \frac{1}{\gamma}\dot{y}(t)\right) \\
 &= G_P e(t) + G_D \dot{e}(t)
 \end{aligned}
 \tag{15}$$

221 to minimize e .

222 The signals, contributing to this type of control are the feedback signals position $y(t)$ and
 223 velocity $\dot{y}(t)$, and the desired time series for $y_{\text{des}}(t)$, and $\dot{y}_{\text{des}}(t)$. The sensor limits for these
 224 signals were taken from the original desired trajectories as $u_{y,\text{min}} = u_{y_{\text{des}},\text{min}} = \min(y_{\text{des}}(t))$
 225 and $u_{y,\text{max}} = u_{y_{\text{des}},\text{max}} = \max(y_{\text{des}}(t))$, and in an analogous manner for the velocity signals.
 226 All four signals were encoded with the same time resolution Δt and amplitude resolutions
 227 $\Delta u_y = \Delta u_{y_{\text{des}}} = (u_{y,\text{max}} - u_{y,\text{min}})/n$, and $\Delta u_{\dot{y}} = \Delta u_{\dot{y}_{\text{des}}} = (u_{\dot{y},\text{max}} - u_{\dot{y},\text{min}})/n$, respectively.
 228 The total processed information I can thus directly be calculated by four addends of type
 229 Eq. 5:

$$\begin{aligned}
 I &= \frac{T}{\Delta t} \left(2 \log_2 \left(1 + \frac{u_{y,\text{max}} - u_{y,\text{min}}}{\Delta u_y} \right) \right. \\
 &\quad \left. + 2 \log_2 \left(1 + \frac{u_{\dot{y},\text{max}} - u_{\dot{y},\text{min}}}{\Delta u_{\dot{y}}} \right) \right) .
 \end{aligned}
 \tag{16}$$

230 This model is the simplest implementation of negative feedback control that allows to enforce
 231 a desired hopping trajectory on a technical system.

232 **Step 3:** Find I_{min} by optimizing the signal resolution parameters. By definition, the in-
 233 formation I depends on the chosen resolutions Δt and Δu (Eq. 5). To adequately compare
 234 the three models, the minimally required information I_{min} for stable hopping was deter-
 235 mined by optimization. For this purpose, hopping was first performed with high resolutions
 236 (equivalent to large I) to determine a reference performance P_{max} . The reference perfor-
 237 mance was verified with sensor resolutions one magnitude higher, which resulted in the same

238 performance P_{\max} . Then, the resolutions, and thus I , were systematically decreased. For
 239 each resolution, the control parameters (feed-forward pattern $A(t)$, or feedback parameters
 240 G , or G_P and G_D , respectively) were optimized for maximum stable hopping performance.
 241 I_{\min} was chosen as the lowest resolution resulting in stable hopping with a hopping perfor-
 242 mance of at least $P_{I_{\min}} > 0.9P_{\max}$. To gain an estimate of the error of I_{\min} , the difference
 243 $\Delta I = I_{\min-1} - I_{\min}$ to the second lowest resolution found in the optimization was calculated.
 244 ΔI thus gives an estimate for a possible reduction in I_{\min} if the search of the optimiza-
 245 tion would be further refined. The optimization step size was chosen such that the error
 246 $\Delta I \approx 10\%I_{\min}$. A corresponding error was calculated for the resolutions and control parame-
 247 ters. These errors specify the steps by which the values were varied during the optimization.

248 In the MFF model, first the time-series $A(t)$ was optimized for hopping performance P
 249 (Eq. 7) on grids with different resolutions ΔA and $\Delta t_{\text{pattern}}$ (algorithm described in [26]).
 250 The pattern with the lowest encoded information was found to have resolutions of $\Delta t_{\text{pattern}} =$
 251 0.125 ± 0.025 s and $\Delta A = 0.125 \pm 0.025$, therefore $I_{\text{pattern}} = 12 \pm 3$ bits. Afterwards, the
 252 minimal required time resolution of the trigger event detection was searched ($\Delta t_{\text{take-off}} =$
 253 0.02 s). The minimal required information to generate hopping in this model was thus found
 254 to be $\mathbf{I_{\min} = (34 \pm 3) \text{ bits}}$. The achieved hopping performance at I_{\min} was $P_{I_{\min}} = 0.062 \text{ m} =$
 255 $91\% \cdot P_{\max}$. Further reduction of the resolution resulted in frequencies $f < 2$ Hz and thus
 256 reduced performance $P < 90\% \cdot P_{\max}$.

257 In the MFB model, the information to be transmitted from the force sensor to the muscle
 258 only depends on the signal resolution parameters Δu and Δt . Their optimization resulted
 259 in $\mathbf{I_{\min} = (32 \pm 2) \text{ bits}}$. The corresponding resolutions of the feedback-signal were $\Delta t_{I_{\min}} =$
 260 0.0650 ± 0.0025 s and $\Delta u_{I_{\min}} = 0.0525 \pm 0.0025$. The control parameter feedback gain was
 261 $G = 9.43 \pm 0.24$. The achieved performance was $P_{I_{\min}} = 0.068 \text{ m} = 96\% \cdot P_{\max}$. As in the
 262 MFF model, further reduction of the resolution resulted in frequencies $f < 2$ Hz and thus
 263 reduced performance $P < 90\% \cdot P_{\max}$.

264 In the EFB model, the minimally required information was found to be $\mathbf{I_{\min} = (660 \pm 63) \text{ bits}}$.
 265 The corresponding resolutions of the PD-controller were $\Delta t_{I_{\min}} = 0.0082 \pm 0.001$ s, $\Delta u_{y, I_{\min}} =$
 266 0.024 ± 0.004 , and $\Delta u_{\dot{y}, I_{\min}} = 0.49 \pm 0.08$. The optimal feedback gains were $G_P =$
 267 $(6.250 \pm 0.25)10^3 \text{ Vm}^{-1}$ and $G_D = (1 \pm 0.1)10^3 \text{ Vsm}^{-1}$ resulting in a hopping performance of
 268 $P_{I_{\min}} = 0.067 \text{ m} = 95\% \cdot P_{\max}$. In this model further reduction of the resolution results in
 269 unstable behavior rather than a reduction in performance.

270 **IV. DISCUSSION**

271 We have presented a new method to quantify *control effort*. Despite – or more likely
 272 because of – its simple form, it can be directly applied to real technical control systems,
 273 but also to models of biological and technical control systems by discretizing all (virtual)
 274 sensor signals contributing to the control. The foundation, Shannon’s information entropy,
 275 is not new. Since its publication in 1948 [46], it has been applied in many fields. Its
 276 general formulation (Eq. (1)) is based on the probabilities $p(x)$ of certain signals, code
 277 words, etc. The application of this general form to real or modeled control systems is
 278 often difficult, as these probabilities are unknown a priori. With the assumption of equal
 279 probability for each sensor value, the application becomes possible (Eq. (4)). Using this
 280 simplification, our approach takes advantage of the fact that information entropy, applied
 281 to sensor measurements, gives a numerical value for the processed information. This value
 282 is – in the sense of optimal control [38] – a cost function allowing to quantify, compare, and
 283 optimize systems. The novelty of our approach is to measure the information entropy of the
 284 movement control process as a function of physical structure and control method:

$$I_{\text{movement}} = I_{\text{movement}}(\text{structure, control}) \quad (17)$$

285 The chosen examples demonstrate this. All three models MFF, MFB, and EFB execute the
 286 same motion, i.e. periodic hopping. The models are of similar reduced complexity with only
 287 one actuator and the minimum number of control signals required. Thus, the results are
 288 comparable. Furthermore, the muscle models take important non-linear characteristics of
 289 biological muscles into account and rely on simple bio-inspired control schemes, while the
 290 technical model implements a standard technical actuator in combination with a standard
 291 feedback controller to generate a desired hopping trajectory. Between muscle models MFF
 292 and MFB, only the control is varied while the bio-physical structure, i.e. the muscle fibers’
 293 material properties, is the same (Eq. (17)). For the electric motor model EFB, additionally
 294 the physical structure is varied. The resulting values for the minimally required information
 295 reveal that the muscle requires less control effort for hopping as it requires considerably
 296 less information to generate and stabilize the movement (MFF: $I_{\min} = 34$ bits, or MFB:
 297 $I_{\min} = 32$ bits), compared to the engineering approach (EFB: $I_{\min} = 660$ bits). This confirms
 298 that the requirements on information processing – or *cognitive load* – depends on the (bio-
 299)physical properties of a control system [52].

300 In contrast to other studies [53, 54], our approach only quantifies the information leading
 301 from sensor signals to actuator actions while ignoring the information back-flow via system
 302 dynamics and environment. Within the sensor signal lies the relevant information for the
 303 controller to generate the desired movement, also called *pragmatic information* [55]. Prag-
 304 matic information is only the information that actually generates a measurable action or
 305 change in structure stripped of all the redundant and unnecessary information. The assump-
 306 tion of equal distribution ($p_i = 1/n$) results in the upper bound of the information compared
 307 to all other possible distributions ($p_i \neq 1/n$) [56]:

$$0 \leq - \sum_{i=1}^n p_i \log_2 p_i \leq \log_2 n .$$

308 Therefore, our approach typically overestimates the transmitted information. The opti-
 309 mization with I as cost function to be minimized by varying sensor resolutions is a way to
 310 approximate the pragmatic information of the sensor signal and thus allowing a comparison
 311 of different realizations of the same movement — the lower I_{\min} is, the lower is the pragmatic
 312 information while I_{\min} with $p_i = 1/n$ gives its upper bound.

313 The more general approach to include the flow of information back to its sensors, that
 314 the actuated system causes via system dynamics and environment, additionally allows to
 315 investigate optimal principles on the decisions which movement to perform [53, 54]. At
 316 the moment, our approach only targets different realizations of the same movement and
 317 focuses on the differences resulting from the (bio-)physical properties of the system. For
 318 this purpose, it is required to implement the method in concrete models of control systems
 319 with the trade-off on the generality of the conclusions. Also, the simplifying assumption of
 320 equal distribution only approximates information theoretic limits. More general conclusions
 321 on information theoretic limits of open-loop (feed-forward) vs. closed-loop (feedback) control
 322 can only be drawn if the models' explicit (bio-)physical properties are not considered and
 323 the probability distribution of the sensor readings are not restricted [43, 44]. Future work
 324 has to reveal the relation of our approximation to the information theoretic limits in the
 325 control loop.

326 Our method to quantify control effort relies on the explicit definition of the desired
 327 movement and a corresponding definition of the movement performance (here, Eq. 7). Stable
 328 hopping is quite suitable for this purpose. The trade-off is, however, that the results are
 329 only valid for just the investigated movement. It is expected that the results may vary

330 quite substantially for other movements . This limitation to general conclusions about
331 control systems is also known from other quantification criteria, e.g. energy requirements,
332 which are movement specific too. The reduced examples on hopping therefore primarily
333 demonstrate the application of our proposed measure. To confirm in general the hypothesis
334 that muscles reduce control effort compared to technical actuators in the control of biology-
335 like movements, more models, movements, and controllers need to be compared.

336 Nevertheless, we expect that more complex biomechanical models will confirm the low
337 control effort requirements. The material properties of the actuator in the muscle driven
338 models MFF and MFB represent typical properties of biological muscles, i.e. the well-
339 known muscle force-length-velocity dependency. These reduced models were specifically
340 introduced to reveal the relevance of muscle properties in periodic movements, such as
341 hopping [26, 47] and confirmed previous findings that the force-length-velocity relation of
342 muscles is significant with respect to the control of biological movements [1, 26, 50, 51, 57, 58].
343 More precisely, neglecting the force-velocity relation results in unstable hopping with the
344 proposed simple bio-inspired controllers in the reduced [26, 47] and in more complex models
345 [50, 51]. We therefore expect that also the tendency for little required control effort will
346 be inherited by more detailed models of human hopping. The reason for this expectation
347 is the concept of *exploitive actuation* [5]. If the mechanical system is well designed, part
348 of the control can be attributed to the mechanical system itself and thus be exploited by
349 simple controllers. There is evidence that biological systems are designed in such a way
350 [5, 24, 52, 59–62]. A technical solution which is not specifically optimized with respect to this
351 concept may require significantly larger control effort. However, we expect that also technical
352 solutions can be found which result in a desired movement with minimal control effort.
353 There are even technical solutions that require less control effort than their biological role
354 model. For example, passive dynamic walkers and runners, which perform stable biology-like
355 locomotion without any actuation [63–65] and therefore with zero control effort ($I_{\text{movement}} =$
356 0). However, passive dynamic walkers or runners can only perform one specific movement. In
357 contrast, humans can perform a large variety of movements, including walking and running.
358 This demonstrates that control effort calculated by information entropy may prove to be an
359 important measure in the study of biological systems. As opposed to previous definitions
360 of control effort, it captures the simplicity aspect of movement control. It is therefore one
361 additional quantitative measure, besides e.g. stability, consumed energy, time to complete a

362 task, minimum jerk, performance, and other possible cost functions, allowing a quantitative
363 comparison of structurally different realizations of the same movement. Further research
364 has to reveal whether minimal processed information is a design principle in nature as some
365 studies suggest [66].

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