Comment on "Nonclassical Paths in the Recurrence Spectrum of Diamagnetic Atoms"

In a recent Letter [1] Granger and Greene discuss the effects of nonclassical paths on the recurrence spectra of diamagnetic atoms and claim an intimate relationship between two types of nonclassical paths, viz., ghost orbits in complex phase space and diffractive orbits that form when the electron scatters from features smaller than its wavelength. The authors claim that the exponentially decaying recurrence amplitude discussed in [2] consists of two pieces that nearly cancel each other: a diffractive orbit and a ghost orbit, both having similar scaled actions that cannot be resolved in the low-resolution Fourier transform of the quantum photoabsorption spectra. It was already noted in a Comment [3] that "there is not enough evidence given in [1] to purport that view." Granger and Greene had published a Reply in response [4].

In this Letter we prove that the physical picture of destructive interference between diffractive and ghost orbits suggested in [1] is not quite conclusive or at least is incomplete. The starting point is the observation that the scaled actions of the two nonclassical paths, although being similar, do not exactly coincide. Therefore, a high-resolution analysis of the spectra must reveal two separate peaks, one associated with the ghost orbit, e.g., the orbit X_1 in [1], and the other associated with a diffractive orbit, e.g., the core-scattered combination orbit $R_1 + V_1^1$ which is topologically similar to the orbit X_1 (see Fig. 4 in [1]). We have calculated a high-resolution recurrence spectrum for the diamagnetic hydrogen atom at scaled energy $\tilde{E} = -0.15$ by harmonic inversion of the quantum photoabsorption spectrum, as described in [5,6]. The photoabsorption spectrum was obtained by numerically exact quantum calculations using a complete Sturmian type basis set. The results are presented in Fig. 1. The classical scaled actions of the real closed orbits, complex ghost orbits, and diffractive orbits, are marked by plus symbols, diamonds, and crosses, respectively. The quantum results, represented by squares, have been obtained by harmonic inversion of the quantum photoabsorption spectrum. It is evident that they are in excellent agreement with the results for the real classical orbits and, in particular, with the ghost orbit X_1 at complex scaled action $S/2\pi \approx 2.4861 - i0.0052$. The amplitudes of the semiclassical and quantum recurrence peaks (not shown in the figure) are also found to agree very well. However, the diffractive orbits (crosses in Fig. 1) *do not have any quantum analogue* in the recurrence spectrum of the hydrogen atom. In particular, the spectrum does not exhibit any contribution of the diffractive orbit $R_1 + V_1^1$ at scaled energy $S/2\pi \approx 2.5518$. (The diffractive orbits are expected, of course, to appear in the recurrence spectra of nonhydrogenic atoms with nonzero quantum defects.) From the analysis it is evident that the exponentially decaying recurrence amplitude associated with the X_1 peak is perfectly and exclusively described by a ghost orbit. It is also clear that the contribution of a diffractive orbit cannot be completely canceled by destructive interference with a ghost orbit because the actions do not exactly coincide. If the mechanism proposed in [1] was true, both the ghost and the diffractive orbits would have to show remaining (small) contributions in the total recurrence spectrum, which is definitely not the case. Thus, the physical interpretation of the nondecaying peaks associated with ghost orbit recurrences in Fig. 1 of [1] remains obscure.

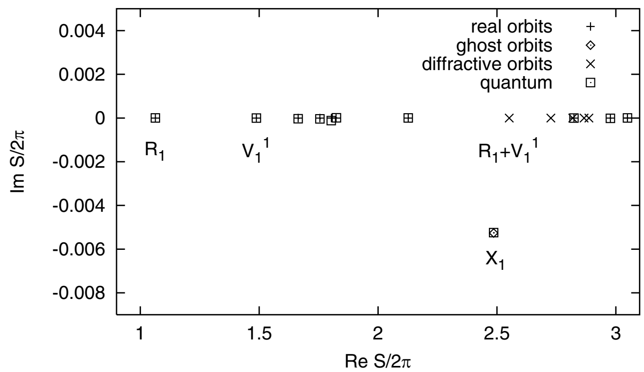

FIG. 1. High-resolution recurrence spectrum of the hydrogen atom in a magnetic field at scaled energy $\tilde{E} = -0.15$. The quantum recurrence peaks (squares) can be associated with the real orbits (plus symbols) and ghost orbits (diamonds). The diffractive orbits (crosses) do not have any quantum analogue in the diamagnetic hydrogen atom.

Jörg Main and Günter Wunner
Institut für Theoretische Physik 1
Universität Stuttgart
D-70550 Stuttgart, Germany

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