

Comment on ‘‘Recurrences without closed orbits’’

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In a recent paper Robiccheaux and Shaw [Phys. Rev. A **58**, 1043 (1998)] calculated the recurrence spectra of atoms in electric fields with nonvanishing angular momentum $\langle L_z \rangle \neq 0$. Features are observed at scaled actions ‘‘an order of magnitude shorter than for any classical closed orbit of this system.’’ We investigate the transition from zero to nonzero angular momentum and demonstrate the existence of short closed orbits with $L_z \neq 0$. The real and complex ‘‘ghost’’ orbits are created in bifurcations of the ‘‘uphill’’ and ‘‘downhill’’ orbit along the electric-field axis, and can serve to interpret the observed features in the quantum recurrence spectra. [S1050-2947(99)05607-3]

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In Ref. [1] Robiccheaux and Shaw calculate quantum photoabsorption spectra of atoms in electric fields with nonzero magnetic quantum numbers, $m \neq 0$ and observe recurrence peaks at short actions in the Fourier transform recurrence spectra. For spectra with magnetic quantum number $m = 0$ these peaks can be directly interpreted as the recurrences of the ‘‘uphill’’ and ‘‘downhill’’ orbit along the electric-field axis. For nonzero angular momenta the authors argue that ‘‘these two orbits are not possible, because L_z is conserved and there is a repulsive $L_z^2/(x^2 + y^2)$ term in the potential.’’ The observed features at short actions are therefore interpreted as ‘‘recurrences without closed orbits.’’ It is the purpose of this comment to demonstrate that the uphill and downhill orbit do not disappear to nowhere at the transition from zero to nonzero angular momentum L_z , but, in contrast, closed orbits with approximately the same short action still exist for $L_z \neq 0$. As will be shown, the orbits along the z axis undergo bifurcations and split into a real and a complex ‘‘ghost’’ orbit. The importance of ghost orbits for the photoabsorption spectra of atoms in a magnetic field has been discussed at length in [2].

For the hydrogen atom in an electric field the Hamiltonian separates in semiparabolical coordinates, $\mu = \sqrt{r+z}$, $\nu = \sqrt{r-z}$, i.e., $H = H_\mu + H_\nu$ with

$$H_\mu = \frac{1}{2} p_\mu^2 - \varepsilon \mu^2 + \frac{\tilde{L}_z^2}{2\mu^2} + \frac{1}{2} \mu^4 = 2Z_1, \quad (1)$$

$$H_\nu = \frac{1}{2} p_\nu^2 - \varepsilon \nu^2 + \frac{\tilde{L}_z^2}{2\nu^2} - \frac{1}{2} \nu^4 = 2Z_2, \quad (2)$$

$Z_1 + Z_2 = 1$, $\varepsilon = EF^{-1/2}$ is the scaled energy, and $\tilde{L}_z = L_z F^{1/4}$ is the scaled angular momentum. F is the electric-field strength. Obviously, for $L_z \neq 0$ the centrifugal barrier does not allow trajectories to start exactly at the origin. However, the shortest closed orbits can easily be derived from the conditions on the time evolution $p_\nu(\tau) = 0$, $\nu(\tau) = \nu_0 = \text{const}$ for the orbits bifurcating from the uphill orbit and $p_\mu(\tau) = 0$, $\mu(\tau) = \mu_0 = \text{const}$ for the orbits bifurcating from the downhill orbit. In the following, we discuss the bifurcation of the uphill orbit, for the downhill orbit similar

results are obtained at low energies $\varepsilon \ll -2.0$. The effective potential $V(\nu) = -\varepsilon \nu^2 + \tilde{L}_z^2/2\nu^2 - \nu^4/2$ has a local minimum for energies $\varepsilon < -\frac{3}{2} \tilde{L}_z^2/3$. The stationary ν motion is obtained from the condition of vanishing derivative $dV(\nu)/d\nu = 0$, yielding

$$2\nu_0^6 + 2\varepsilon \nu_0^4 + \tilde{L}_z^2 = 0. \quad (3)$$

Two approximate solutions of Eq. (3) at $\varepsilon \ll 0$ are $\nu_0^2 \approx \pm \tilde{L}_z / \sqrt{-2\varepsilon}$ approaching $\nu_0 = 0$ in the limit $\tilde{L}_z \rightarrow 0$. The two solutions represent a real and a ghost orbit for ν_0 real and imaginary, respectively. With the given ν_0 , it is a straightforward task to calculate the constant of motion $Z_1 = 1 - Z_2$ and to solve for $\mu(\tau)$ in Eq. (1). The shapes of the closed orbits are presented in dimensionless scaled coordinates ($\tilde{\rho} = \rho F^{1/2}$, $\tilde{z} = z F^{1/2}$) in Fig. 1 for $\varepsilon = -4.0$ and scaled angular momentum $\tilde{L}_z = 0.014$, which belongs to the magnetic quantum number $m = 1$ at a value $\omega = 2\pi \sqrt{\varepsilon/E} = 450$. This corresponds to the values chosen in Fig. 1 of Ref. [1]. The solid line is the real orbit, which is the uphill orbit distorted by the

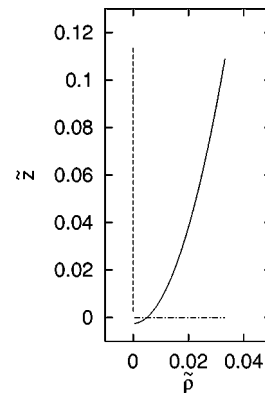


FIG. 1. Closed orbits of the hydrogen atom in an electric field at scaled energy $\varepsilon = -4.0$ and angular momentum $\tilde{L}_z = 0.014$ drawn in dimensionless scaled coordinates ($\tilde{\rho} = \rho F^{1/2}$, $\tilde{z} = z F^{1/2}$). Solid line: Real orbit. Dashed and dash-dotted lines: Real and imaginary part of the complex ‘‘ghost’’ orbit, respectively. The orbits have bifurcated from the uphill orbit at vanishing angular momentum and have scaled action $\tilde{S} \approx (-2\varepsilon)^{-1/2} = 0.35$.

repulsive centrifugal barrier. An analogous orbit has been discovered and discussed for the diamagnetic Kepler system with nonvanishing angular momentum [3]. The dashed and dash-dotted lines are the real and imaginary part of the complex ghost orbit, respectively. The real part of the ghost orbit is nearly identical with the uphill orbit at vanishing angular momentum $\tilde{L}_z=0$. The scaled action of both orbits is $\tilde{S} \approx 1/\sqrt{-2\varepsilon}=0.35$ in perfect agreement with the first recurrence peak in Figs. 2 and 3 of Ref. [1].

As mentioned above, for $L_z \neq 0$ the centrifugal barrier does not allow trajectories to start exactly at the origin. The nearest distance of closed orbits from the origin depends on the values of the constants of motion Z_1 and Z_2 in Eqs. (1) and (2). It is negligibly small for orbits with $Z_1 \approx Z_2 \approx 0.5$ and increases when Z_1 or Z_2 approaches the minimal allowed value. For the real closed orbit in Fig. 1 the nearest distance from the origin is $\tilde{r}_{\min} = r_{\min} F^{1/2} = 0.0025$ in scaled units, which is about 13 Bohr radii at $\omega = 2\pi\sqrt{\varepsilon/E} = 450$. This is slightly outside the classically allowed region of the initial state $|2p1\rangle$, however, it should be noted that a small change of the initial conditions results in approximately closed orbits where the distance to the origin at the start and return is reduced to about a few Bohr radii. The real closed orbit in Fig. 1 is more strongly excited in dipole transitions from initial states of larger size than the size of the hydrogenic $|2p1\rangle$ state as can clearly be seen in Figs. 5 and 6 of Ref. [1] for the excitation of the K and Cs atom, respectively. The relatively large nearest distance of the closed orbit to the

origin suppresses effects of classical core scattering [4], especially for the K atom (see Fig. 5 in Ref. [1]), which might result in strong damping of the multiple repetitions for orbits with $L_z=0$ diving deeply into the ionic core. For the Cs atom the ionic core has larger size and core scattering can be observed in Fig. 6 of Ref. [1], but, in contrast to the interpretation given in [1], orbits are scattered into the *truly existing* short closed orbit presented in Fig. 1. However, it is still an outstanding task to reproduce quantitatively the amplitudes of recurrence peaks in the Fourier transform quantum spectra of the hydrogen atom and nonhydrogenic atoms by application of closed orbit theory. Robicheaux and Shaw are probably right that the theory may need to be generalized to account for the effects of orbits not starting at and returning back exactly to the origin.

In conclusion, we have investigated the bifurcation scenario of the shortest closed orbits of the hydrogen atom in an electric field at the transition from zero to nonvanishing angular momentum L_z , and revealed the existence of short real and complex ghost orbits with $L_z \neq 0$. They are born in bifurcations of the uphill and downhill orbit along the field axis and can serve to interpret features at short scaled actions in the quantum recurrence spectra calculated by Robicheaux and Shaw [1].

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